A Servicing Strategy Involving Imperfect Repair For Products Sold with One-Dimensional Warranties

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Abstract

The warranty period offered by a manufacturer tends to be longer ranging from 3 to 7 years. Offering a product with a longer warranty period increases the warranty cost and this needs an effort to reduce it. For a repairable product, servicing strategies which combine minimal repair and imperfect repair can reduce the warranty cost significantly. In these servicing strategies, the number of imperfect repairs is only one over the warranty period. For a longer warranty period, more imperfect repairs would be needed in order to reduce the number of failures over the warranty period. Varnosafaderani and Chukova [1] studied a servicing strategy which can carry out imperfect repairs more than one for products sold with two-dimensional warranties. In this strategy, imperfect repairs are restricted in the middle regions of warranty period. In this paper we study servicing strategy where imperfect repairs are unrestricted by middle regions (middle intervals for one-dimensional warranties) but dependent on the age of failure under warranty. A numerical example is given to illustrate the optimal solution and compare this strategy with the existing similar servicing strategy that has been studied previously.

Keywords
Servicing strategy, imperfect repair, one-dimensional warranty, expected warranty cost

1. Introduction

In most market place, the warranty period offered by a manufacturer tends to be longer for example products of electronics and automotives are sold with warranty period ranging from 3 to 7 years. Offering a product with a longer warranty period increases the warranty cost to the manufacturer and this becomes a major interest to reduce it. For a repairable product, an appropriate servicing strategy can signifantly reduce the warranty cost [4].

Servicing strategies which have been developed in the literature can be categorized into products sold with one-dimensional warranties e.g. Jack and Van der Duyn Schouten [2], Jack and Murthy [3] and Yun et al. [4] and two-dimensional warranties e.g. Varnosafaderani and Chukova [1] and Iskandar and Jack [5]. Yun et al. [4] and Iskandar and Jack [5] have shown that servicing strategies which involve minimal repair and imperfect repair can reduce the warranty cost significantly. In these servicing strategies, the number of imperfect repairs is only one over the warranty period. For a longer warranty period, more imperfect repairs would be needed in order to reduce the number of failures over the warranty period.

Varnosafaderani and Chukova [1] studied a servicing strategy allowing more than one imperfect repairs for products sold with two-dimensional warranties. The warranty period is divided by several regions (consisting of first region, middle regions, and last region) and each first failure occurring in a region belonging to the middle regions is fixed by imperfect repair while all other failures are fixed by minimal repair. For the case of a product sold with a one-dimensional warranty, the warranty period is divided by several intervals and imperfect repairs are done with the same fashion. In this strategy, imperfect repairs are restricted in the middle regions or intervals (i.e. the second and/or the third intervals) of warranty period. An alternative servicing strategy is the one where imperfect repairs are based on the age at failure.

In this paper we study a servicing strategy for a repairable product sold with a one-dimensional warranty where imperfect repair can be done more than one time under warranty. This can be viewed as the extension of the servicing strategy developed by Iskandar et al. [5] to the case of imperfect repair instead of replacement. Under this strategy, imperfect repair is done at failure if the elapsed time since the last imperfect repair (or the beginning of the operation, $t = 0$) is greater than τ (a threshold value).

The outline of the paper is as follows. Section 2 describes of the model formulation. Section 3 deals with the model analysis. In section 4, we give a numerical example to illustrate the optimal solution and compare the solution of this strategy with those of similar servicing strategies have been studied in the literature. Finally, we conclude with a brief discussion of a few topics for future research.

2. Model Formulation

The following notation is used in model formulation.

Notation

W	: warranty period (in year)
δ	: improvement level in imperfect repairs ($0 < \delta < 1$)
W'	: time limit where all failures in $(W', W]$ are minimally repaired in Strategy 1 ($0 \le W' \le W$)
τ	: threshold value for imperfect repairs in Strategy 1 ($0 \le \tau \le W'/2$)
W_1, W_2, W_3	: interval limit in Strategy 2 ($0 \le W_1 \le W_2 \le W_3 \le W$)
$A_1(t)$, $A_2(t)$	: virtual age of the item after the first and second imperfect repair, respectively
$F(t)$, $f(t)$, $h(t)$	: failure distribution, density and hazard rate functions
S_1, S_2	: the first failure after τ and the first failure after $S_1 + \tau$, with distribution function $F_1(s_1)$ and $F_2(s_2)$, respectively
$f_1(s_1)$, $f_2(s_2)$	: density function associated with $F_1(s_1)$ and $F_2(s_2)$, respectively
$h_1(t)$, $h_2(t)$	: hazard rate function associated with $F_1(s_1)$ and $F_2(s_2)$, respectively
c_m	: cost of minimal repair
c_p	: cost of perfect repair
$c_i(\delta)$	: cost of imperfect repair as a function of δ
$J_1(\delta, \tau, W')$	: expected warranty servicing cost of Strategy 1
$J_2(\delta, W_1, W_2, W_3)$	: expected warranty servicing cost of Strategy 2

2.1 Servicing Strategy

We consider a repairable product sold with a one-dimensional free repair warranty with warranty period W. Two servicing strategies–namely Strategy 1 and Strategy 2 will be studied.

Strategy 1:

Imperfect repair is done at failure (at time t, $t \le W' \le W$) if the elapsed time since the last imperfect repair (or the beginning of the operation, $t = 0$) is greater than τ (a threshold value). All other failures are fixed by the minimal repair. As a result, this servicing strategy allows more than one imperfect repair.

Strategy 2:

The warranty period is divided by four intervals i.e. $(0, W_1]$, $(W_1, W_2]$, $(W_2, W_3]$ and $(W_3, W]$. Each first failure in interval $(W_1, W_2]$ and $(W_2, W_3]$ is imperfectly repaired with improvement level δ and all other failures are minimally repaired.

Strategy 2 is the servicing strategy by Varnosafaderani and Chukova [1] for the one-dimensional case. Fig.1 gives the illustration of imperfect repairs done under the two strategies. We consider that imperfect repair improves the reliability of the item–by reducing the age of the item. Each imperfect repair will result in reducing the age with improvement level δ .

Strategy 1

Strategy 2

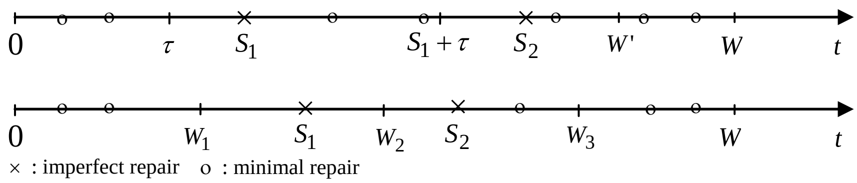

Figure 1. Strategy 1 and 2 with two imperfect repairs

2.2 Effect of Imperfect Repairs

Imperfect repairs improve the reliability of the repaired item in the sense that the hazard rate of the item after repair is smaller than that before failure. The effect of imperfect repair can be modelled through either hazard rate or age reduction models [6]. In this paper we use the age reduction model and it is described as follows. We model the effect of the imperfect repairs by reducing the virtual age of the repaired item. Let $A(t)$ be the virtual age of the item at time *t*. The hazard rate of the item is as function of $A(t)$, denoted by $h(A(t))$. If the imperfect repair is done at age s_1 with improvement level δ then the virtual age and the hazard rate after repair are given by $A_1(t) = t - \delta s_1$ and $h_1(t) = h(t - \delta s_1)$ for $t > s_1$. For the second imperfect repair occurring at age s_2 the virtual age and hazard rate are given by $A_2(t) = t - \delta s_1 + \delta^2 s_1 - \delta s_2$ and $h_2(t) = h(t - \delta s_1 + \delta^2 s_1 - \delta s_2)$ for $t > s_2$ **,** respectively. For the case where all failures are rectified by minimal repairs then $A(t)$ is equal to t that is called actual age.

2.3 Modeling Failures for Imperfect Repairs

We consider the case where the number of imperfect repairs over the warranty period is at most two times. Let S_1 and S_2 denote the first failure after τ and the first failure after $S_1 + \tau$, respectively. The distribution functions for S_1 and S_2 [$F_1(s_1)$ and $F_2(s_2)$] are given as follows. As failures occurring in $(0, \tau]$ and $(S_1, S_1 + \tau]$ are fixed by minimal repair, then

$$F_1(s_1) = 1 - \exp[H(\tau) - H(s_1)] \tag{1}$$

where $H(t) = \int_0^t h(u)du$.

Differentiating (1) with respect to s_1 yields

$$f_1(s_1) = h(s_1)\exp[H(\tau) - H(s_1)] . \tag{2}$$

Conditioning on $S_1 = s_1$ and then unconditioning it, we have $F_2(s_2)$ given by

$$F_2(s_2) = \int_\tau^{W'} \{1 - \exp[H_1(\tau + s_1) - H_1(s_2)]\} h(s_1)\exp[H(\tau) - H(s_1)]ds_1 \tag{3}$$

where $H_1(t) = \int_0^t h_1(u)du$ and its density function is given by

$$f_2(s_2) = \int_\tau^{W'} h_1(s_2)\exp[H_1(\tau + s_1) - H_1(s_2)] h(s_1)\exp[H(\tau) - H(s_1)]ds_1 . \tag{4}$$

Distribution functions of S_1 and S_2 for Strategy 2 can be obtained by converting the corresponding distribution functions in Varnosafaderani and Chukova [1] into one-dimensional case.

3. Model Analysis

For an item covered by a longer warranty period (e.g. 5 to 7 years), it may require a servicing strategy to involve more than one imperfect repair over the warranty period in order to reduce the warranty cost. In the servicing strategy developed, we consider to the case where the number of imperfect repairs during the warranty period at most N times, where $N = 2$.

3.1 Expected Warranty Servicing Cost

The expression for the expected warranty servicing cost $J_1(\delta, \tau, W')$ for Strategy 1 is obtained by a conditional approach. Define S_1 and S_2 as in Section 2. The number of imperfect repairs done in the warranty period depends on the values of S_1 and S_2. It could be 0, 1 or 2 imperfect repairs over the warranty. As a result, conditional on $S_1 = s_1$ and $S_2 = s_2$, the expected warranty servicing cost for Strategy 1, $J_1(, \delta, \tau, W' | S_1 = s_1, S_2 = s_2)$ is given by

$$J_1(\delta,\tau,W'|S_1=s_1,S_2=s_2)=\begin{cases} c_m\left[H(\tau)+H(W)-H(W')\right] & if \quad s_1,s_2>W' \\ c_i(\delta)+c_m\left[H(\tau)+H_1(W')-H_1(s_1)\right] & if \quad W'-\tau<s_1\le W',s_2>W' \\ c_i(\delta)+c_m\left[H(\tau)+H_1(s_1+\tau)-H_1(s_1)+H_1(W)-H_1(W')\right] & if \quad \tau<s_1\le W'-\tau,s_2>W' \\ 2c_i(\delta)+c_m\left[H(\tau)+H_1(s_1+\tau)-H_1(s_1)+H_2(W)-H_2(s_2)\right] & if \quad \tau<s_1\le W'-\tau,s_2\le W' \end{cases} \tag{5}$$

where $H_2(t) = \int_0^t h_2(u)du$.

Removing the conditioning in (5) yields

$$\begin{aligned} J_1(\delta,\tau,W') = {} & c_m\left[H(\tau)+H(W)-H(W')\right]\exp[H(\tau)-H(W')] + \\ & \int_{W'-\tau}^{W'}\left\{c_i(\delta)+c_m\left[H(\tau)+H_1(W)-H_1(s_1)\right]\right\}h(s_1)\exp[H(\tau)-H(s_1)]ds_1 + \\ & \int_{\tau}^{W'-\tau}\left\{c_i(\delta)+c_m\left[H(\tau)+H_1(s_1+\tau)-H_1(s_1)+H_1(W)-H_1(W')\right]\right\} \\ & \exp[H_1(s_1+\tau)-H(W')]h(s_1)\exp[H(\tau)-H(s_1)]ds_1 + \\ & \int_{\tau}^{W'-\tau}\int_{s_1+\tau}^{W'}\left\{2c_i(\delta)+c_m\left[H(\tau)+H_1(s_1+\tau)-H_1(s_1)+H_2(W)-H_2(s_2)\right]\right\} \\ & h_1(s_2)\exp\left[H_1(\tau+s_1)-H_1(s_2)\right]h(s_1)\exp[H(\tau)-H(s_1)]ds_2ds_1. \end{aligned} \tag{6}$$

Using the similar approach as in Strategy 1 we have the expected warranty servicing cost for Strategy 2 $J_2(\delta, W_1, W_2, W_3)$ given by

$$\begin{aligned} J_2(\delta,W_1,W_2,W_3) = {} & c_m\left[H(W_1)+H(W)-H(W_3)\right]\exp[H(W_1)-H(W_3)] + \\ & \int_{W_2}^{W_3}\left\{c_i(\delta)+c_m\left[H(W_1)+H_1(W)-H_1(s_1)\right]\right\}h(s_1)\exp[H(W_1)-H(s_1)]ds_1 + \end{aligned}$$

$$\int_{W_1}^{W_2}\{c_i(\delta)+c_m[H(W_1)+H_1(W_2)-H_1(s_1)+H_1(W)-H_1(W_3)]\}$$
$$\exp[H_1(W_2)-H_1(W_3)]h(s_1)\exp[H(W_1)-H(s_1)]ds_1+$$
$$\int_{W_1}^{W_2}\int_{W_2}^{W_3}\{2c_i(\delta)+c_m[H(W_1)+H_1(W_2)-H_1(s_1)+H_2(W)-H_2(s_2)]\}$$
$$h_1(s_2)\exp[-H_1(s_2)+H_1(W_2)]h(s_1)\exp[H(W_1)-H(s_1)]ds_2ds_1. \quad (7)$$

3.2 Optimization

For Strategy 1, the optimization problem is $\min_{\delta,\tau,W'} J_1(\delta,\tau,W')$ subject to the constraints $0\le\tau\le W'/2$ and $0\le W'\le W$, where $J_1(\delta,\tau,W')$ is given by the integral equation (6). We use a numerical search to obtain the optimal values τ^*, W'^* and δ^*. A numerical approach is also needed to obtain the optimal parameter values δ^*, W_1^*, W_2^* and W_3^* for Strategy 2.

4. Numerical Example

We consider $F(t)$ is given by Weibull distribution with two parameters - α and β (representing scale and the shape parameters, respectively). We use the following nominal parameter values: $\alpha=3(years)$, $\beta=2$, $c_m=1$, and $W=7$ years. The cost of imperfect repair is considered as a function of δ given by $c_i(\delta)=c_m+(c_p-c_m)\delta^4$ as in [4]. Table 1 shows optimal solutions for Strategy 1 for a variety of c_p and a fixed value of δ.

Table 1. The optimal solutions for Strategy 1, c_p = 2, 4, ..., 10, and $\delta=0.5$

c_p	$c_i(\delta)$	τ^*	W'^*	$J_1(\tau^*,W'^*)$
2	1.06	0.77	6.89	3.760
4	1.19	0.84	6.66	3.993
6	1.31	0.94	6.41	4.220
8	1.44	1.06	6.14	4.440
10	1.56	1.22	5.86	4.649

Remarks: τ^* increases and W'^* decreases as c_p increases. Meaning that increasing c_p (and hence $c_i(\delta)$) makes τ^* larger in order to gain a bigger benefits of imperfect repairs done.

Now we compare the performances of Strategy 1 and Strategy 2. Two conditions are considered. We first consider δ as a parameter (or a fixed value) and then δ as a decision variable. Tables 2 and 3 show the results for a fixed value of δ and for δ as a decision variable, respectively.

Table 2. Results of Strategy 1 and 2 for $\delta=0.5$

		Strategy 1			Strategy 2			
c_p	$c_i(\delta)$	τ^*	W'^*	J_1^*	W_1^*	W_2^*	W_3^*	J_2^*
2	1.06	0.77	6.89	**3.760**	1.43	3.98	6.90	3.812
4	1.19	0.84	6.66	**3.993**	1.49	3.90	6.70	4.026
6	1.31	0.94	6.41	**4.220**	1.57	3.82	6.47	4.235
8	1.44	1.06	6.14	4.440	1.66	3.73	6.34	**4.438**
10	1.56	1.22	5.86	4.649	1.78	3.63	5.98	**4.634**

Table 3. Results of Strategy 1 and 2 for δ as a decision variable

	Strategy 1					Strategy 2					
c_p	$c_i(\delta)$	δ^*	τ^*	W'^*	J_1^*	$c_i(\delta)$	δ^*	W_1^*	W_2^*	W_3^*	J_2^*
2	1.25	0.71	1.01	6.62	**3.536**	1.28	0.73	1.38	3.80	6.65	3.559
4	1.20	0.51	0.86	6.63	**3.992**	1.22	0.52	1.49	3.88	6.64	4.021
6	1.19	0.44	0.81	6.63	**4.175**	1.21	0.45	1.54	3.90	6.64	4.205
8	1.18	0.40	0.77	6.63	**4.286**	1.18	0.40	1.56	3.91	6.64	4.315
10	1.17	0.37	0.75	6.63	**4.363**	1.17	0.37	1.58	3.92	6.63	4.393

Table 2 shows that Strategy 1 is a better cost for c_p = 2, 4, and 6 and Strategy 2 is a better cost for c_p = 8, and 10. But if δ is viewed as a decision variable, then Strategy 1 always gives a lower cost (See Table 3). Table 4 shows the effect of the scale parameter α (or the item reliability) and the length of the warranty period to the optimal solutions for the two strategies (note that $c_m = 1$, $\beta = 2$, and $c_p = 6$).

Table 4. The best strategy between Strategy 1 and 2 versus on α and W

α	MTTF	W (year)		
		3	5	7
1.0	0.89	SS-2	SS-2	SS-2
1.5	1.33	SS-1	SS-2	SS-2
2.0	1.77	SS-1	SS-1	SS-2
2.5	2.22	SS-1	SS-1	SS-1
3.0	2.66	SS-1	SS-1	SS-1
3.5	3.10	SS-1	SS-1	SS-1
4.0	3.54	SS-1	SS-1	SS-1

Table 4 identifies the lowest cost strategy for different values of α and W. Strategy 2 (SS-2) is the best when the item reliability is low for each value of W. When the item reliability increases (as the value of α increases), the best strategy is Strategy 1 (SS-1).

In Strategy 2, the time between two imperfect repairs can be very small (less than τ) and this will give a better cost for an item with low reliability. For a high reliability item $(\alpha \geq 2.5)$, Strategy 1 is always the best strategy.

5. Conclusions

In this paper we study a servicing strategy that involves imperfect repair for a repairable product sold with a one-dimensional warranty. Imperfect repair done is dependent on the age of failure in which the criteria used is different with that in Varnosafaderani and Chukova [1]. The strategy developed (Strategy 1) is always the best strategy for a high reliability whilst Strategy 2 for the low reliability. Moreover, Strategy 1 is easier to implement as it is only required to record the elapsed time since the last imperfect repair and τ (and hence a simple administrative work) in deciding an imperfect repair.

Topics for future research are described as follows. The servicing strategy developed allows at most two imperfect repairs. The realistic servicing strategy is one which allows more than two imperfect repairs under warranty. The other topic is to extend the servicing strategy to the two-dimensional warranties. These topics are currently under investigation.

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