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Optimal Servicing Strategy Involving Imperfect Repair and Preventive Maintenance for Products Sold with One-Dimensional Warranties

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ABSTRACT

A manufacturer tends to offer a longer warranty period ranging from 3 to 7 years in order to get its product more competitve in a market place. Offering a product with a longer warranty period would increase the warranty cost and this is of great interest to the manufaturer to reduce it. For a repairable product, an effective servicing strategy which combines minimal repair and imperfect repair can reduce the warranty cost significantly. But this servicing strategy confines the number of imperfect repairs, at most one, over the warranty period. For a longer warranty period, more imperfect repairs would be needed in order to reduce the number of failures over the warranty period. In this paper we study two new servicing strategies where performing of imperfect repairs is dependent on the age of failure under warranty. A numerical example is given to illustrate the optimal solution and compare results of the strategies developed with the similar servicing strategy studied previously.

Keywords: Warranty, servicing strategy, imperfect repair.

Mathematics Subject Classification: 60G55, 90B25.

Computing Classification System: G.1.5.

1. INTRODUCTION

Nowadays, a manufacturer offers a longer warranty period in order to make its products more competitive. For instance, electronic products are warrantied for 3 to 7 years. Offering a product with a longer warranty period would increase the warranty cost to the manufacturer. There are several ways to reduce the warranty cost –i.e. (i) reliability improvement, (ii) burn-in test, (iii) servicing strategy, and (iv) preventive maintenance. For a repairable product, an appropriate servicing strategy can significantly reduce the warranty cost [4].

Servicing strategies can be categorized based on whether a product is sold with one-dimensional warranties i.e. Jack and Van der Duyn Schouten [2], Jack and Murthy [3] and Yun et al. [4] or with two-dimensional warranties i.e. Iskandar and Jack [5] and Varnosafaderani and Chukova [1]. Yun et al. [4] and Iskandar and Jack [5] have shown that servicing strategies which involve minimal repair and imperfect repair can reduce the warranty cost significantly. But [4] and [5] consider only one imperfect repairs over the warranty period. For a longer warranty period, Iskandar et.al. [8] have developed two servicing strategies which allow more than one replacement during the warranty period. The servicing strategies studied replace a failed item by a new item if its age at failure is greater than a threshold value and these strategies are the best strategy for a longer warranty period. If the replacement cost is very expensive than imperfect repairs will be an economical option. Varnosafaderani and Chukova [1] studied a servicing strategy allowing more than one imperfect repairs for products sold with two-dimensional warranties. The warranty period is divided by several regions (consisting of first region, middle regions, and last region) and each first failure occurring in a region belonging to the middle regions is fixed by imperfect repair while all other failures are fixed by minimal repair. For a one-dimensional warranty case, the warranty period is divided by several intervals instead of several regions. In this strategy, imperfect repairs are restricted in the middle regions or intervals of the warranty. An alternative servicing strategy is the one where imperfect repairs are performed based on the age at failure. Iskandar et al. [5] have studied a servicing strategy where imperfect repair is done at failure if the age is greater than τ (a threshold value), otherwise it is minimally repaired. However, the paper only allows one imperfect repair over the warranty period.

In this paper we extend the servicing strategy in Iskandar et al. [5], which allows more than one imperfect repairs under warranty. In this servicing strategy, imperfect repair is done at failure if the elapsed time since the last imperfect repair (or the beginning of the operation) is greater than a threshold value. Furthermore, we will integrate the proposed servicing strategy and a preventive maintenance to form a new strategy in order to get more reduction in the warranty cost. This strategy is appropriate for warranted products where PM is offered one package with the warranty (e.g. an air conditioning system).

The outline of the paper is as follows. Section 2 describes of the model formulation. Section 3 deals with the model analysis. In section 4, we give a numerical example to illustrate the optimal solution and compare the solution of this strategy with those of similar servicing strategies have been studied in the literature. Finally, we conclude with a brief discussion of a few topics for future research.

2. MODEL FORMULATION

The following notation is used in model formulation.

Notation

W	: warranty period (in year)
δ	: improvement level in imperfect repairs ($0<\delta<1$)
W'	: time limit where all failures in $(W',W]$ are minimally repaired in Strategy 1 ($0<W'<W$)

τ : threshold value for imperfect repairs in Strategy 1 ($0<\tau<W'$)
W_1, W_2, W_3 : interval limit in Strategy 2 ($0<W_1<W_2<W_3<W$)
$A_1(t)$, $A_2(t)$: virtual age of the item after the first and second imperfect repair, respectively
$F(t)$, $f(t)$, $h(t)$: failure distribution, density and hazard rate functions
S_1, S_2 : the first failure after τ and the first failure after $S_1+\tau$, with distribution function $F_1(s_1)$ and $F_2(s_2)$, respectively
$f_1(s_1), f_2(s_2)$: density function associated with $F_1(s_1)$ and $F_2(s_2)$, respectively
$h_1(t)$, $h_2(t)$: hazard rate function associated with $F_1(s_1)$ and $F_2(s_2)$, respectively
c_m : cost of minimal repair
$c_{im}(\delta)[c_{ip}(\delta)]$:cost of imperfect repair [imperfect PM] as a function of δ
$J_1(\delta,\tau,W')$: expected warranty servicing cost of Strategy 1
$J_2(\delta,W_1,W_2,W_3)$: expected warranty servicing cost of Strategy 2
$J_3(\delta,S_1,S_2,W_1,W_2)$: expected warranty servicing cost of Strategy 3

2.1. Servicing Strategy

We consider a repairable product sold with a one-dimensional free repair warranty with warranty period W. Three servicing strategies–namely Strategies 1-3 will be studied.

Strategy 1: Imperfect repair is done at failure (at time t, $t \le W' \le W$) if the elapsed time since the last imperfect repair (or the beginning of the operation) is greater than a threshold value τ. All other failures are fixed by a minimal repair. As a result, this servicing strategy does not constraint the number of imperfect repairs in $(0,W')$. The failed product is imperfectly repaired as long as failure occurs in $(0,W')$ and the age since the last imperfect repair is greater then τ.

For the case where imperfect repairs done two times during warranty, let s_1 and s_2 denote the first failure after τ and the first failure after $S_1+\tau$, respectively (See Fig.1). Failures at times S_1 and s_2 result in imperfect repair, since the age at S_1 and the age since the first imperfect repair, (s_2-s_1) are both greater than τ.

Strategy 2: The warranty period is divided into four intervals i.e. $(0,W_1]$, $(W_1,W_2]$, $(W_2,W_3]$ and $(W_3,W]$. The first failure in $(W_1,W_2]$ or in $(W_2,W_3]$ is imperfectly repaired with improvement level δ and all other failures are minimally repaired.

Strategy 2 is the servicing strategy introduced by Varnosafaderani and Chukova [1] for the one-dimensional case. Fig.1 gives the illustration of imperfect repairs done under Strategies 1 and 2. We consider that imperfect repair improves the reliability of the item–by reducing the age of the item. Each imperfect repair will result in reducing the age with improvement level δ.
Strategy 1

Strategy 2

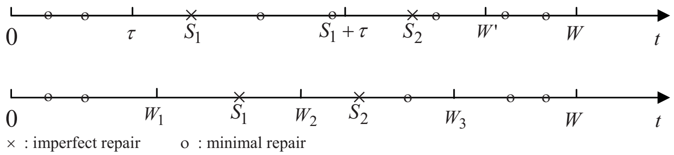

Figure 1. Strategy 1 and 2 with two imperfect repairs

Strategy 3: This strategy is the extention of Strategy 1, which incorporates PM into the strategy or it is a more general servicing strategy. In other words, the proposed strategy integrates PM and servicing strategy to reduce the number of failures and hence it minimises the warranty cost. Under this strategy, the warranty period is divided into five intervals i.e. $(0,S_1]$, $(S_1,W_1]$, $(W_1,S_2]$, $(S_2,W_2]$ and $(W_2,W]$. Each first failure in $(S_1,W_1]$ or $(S_2,W_2]$ is imperfectly repaired and all other failures are minimally repaired. When there is no failure in interval $(S_1,W_1]$ ($(S_2,W_2]$) then PM is done at W_1 (W_2). As metioned before, this strategy is appropriate for the case where the manufacturer offers warranty and PM in one package (e.g. an air conditioning system) in order to give a full protection to the customer under the warranty, and a strong impact to promote the products.

Strategy 3:

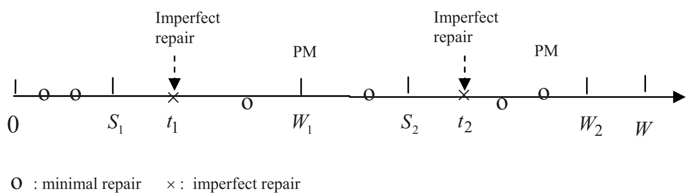

Fig. 2. Servicing strategy with imperfect repairs and PM (Servicing strategy 3)

2.2. Effect of Imperfect Repairs

Imperfect repairs improve the reliability of the repaired item in the sense that the hazard rate of the item after repair is smaller than that before failure. The effect of imperfect repair can be modelled through either hazard rate or age reduction models (Doyen and Gaudion, [6]). In this paper we use the age reduction model and it is described as follows. The effect of the imperfect repair is modelled by reducing the virtual age of the repaired item. Let $A(t)$ be the virtual age of the item at time *t*. The hazard rate of the item is as function of $A(t)$, denoted by $h(A(t))$. If a imperfect repair is done at age s_1 with improvement level δ then the virtual age and the hazard rate after repair are given by $A_1(t)=t-\delta s_1$ and $h_1(t)=h(t-\delta s_1)$ for $t>s_1$. For the second imperfect repair occurring at age s_2 the virtual age and hazard rate are given by $A_2(t)=t-\delta s_1+\delta^2 s_1-\delta s_2$ and $h_2(t)=h(t-\delta s_1+\delta^2 s_1-\delta s_2)$

for $t > s_2$, respectively. Note that if all failures are rectified by minimal repairs then $A(t)$ is equal to t that is called the actual age.

2.3. Modeling Failures for Imperfect Repairs

We consider the case where the number of imperfect repairs over the warranty period is at most two times. Let S_1 and S_2 denote the first failure after τ and the first failure after $S_1 + \tau$, respectively. The distribution functions for S_1 and S_2 [$F_1(s_1)$ and $F_2(s_2)$] are given as follows. As failures occurring in $(0,\tau]$ and $(S_1, S_1+\tau]$ are fixed by minimal repair, then

$$F_1(s_1) = 1 - \exp[H(\tau) - H(s_1)] \tag{1}$$

where $H(t) = \int_0^t h(u)du$.

Differentiating (1) with respect to s_1 yields the density function of S_1 given by

$$f_1(s_1) = h(s_1)\exp[H(\tau) - H(s_1)]. \tag{2}$$

We now obtain the distribution function of S_2. Conditioning on $S_1 = x$ we have

$F_2(s_2 \left| s_1 = x\right.) = \left\{1 - \exp\left[H_1(\tau + x) - H_1(s_2)\right]\right\}$ then unconditioning it, we have $F_2(s_2)$ given by

$$F_2(s_2) = \int_\tau^{W'} \{1 - \exp[H_1(\tau + s_1) - H_1(s_2)]\} h(s_1)\exp[H(\tau) - H(s_1)]ds_1 \tag{3}$$

where $H_1(t) = \int_0^t h_1(u)du$ and its density function is given by

$$f_2(s_2) = \int_\tau^{W'} h_1(s_2)\exp[H_1(\tau + s_1) - H_1(s_2)] h(s_1)\exp[H(\tau) - H(s_1)]ds_1 . \tag{4}$$

Distribution functions of S_1 and S_2 for Strategy 2 can be obtained by converting the corresponding distribution functions in Varnosafaderani and Chukova [1] into one-dimensional case.

2.4. Effect of Imperfect Maintenances

It is assumed that imperfect PM and imperfect repair improve the reliability of the product in the sense that the age of the item after repair is smaller than that before failure. The imperfect PM (imperfect repair) will reduce the age of the product with improvement level δ_1 (δ_2). Without losing the generality of improvement levels, it is considered that $\delta_1 = \delta_2 = \delta$.

We assume that the cost of imperfect PM is smaller than that of imperfect repair under the same improvement level. The cost of imperfect PM is given by $c_{ip}(\delta) = rc_{im}(\delta),\ \ 0 < r < 1$ where r is the cost ratio between the imperfect PM cost and the imperfect repair cost.

3. MODEL ANALYSIS

In the servicing strategies developed, we consider the case where the number of imperfect repairs during the warranty period at most two times.

3.1. Expected Warranty Servicing Cost

The expression for the expected warranty servicing cost $J_1(\delta,\tau,W')$ for Strategy 1 is obtained by a conditional approach. Define S_1 and S_2 as in Section 2. The number of imperfect repairs in the warranty period depends on the values of S_1 and S_2. There are three possible number of imperfect repairs over the warranty, i.e. 0, 1 or 2 imperfect repairs. As a result, conditional on $S_1 = s_1$ and $S_2 = s_2$, the expected warranty servicing cost for Strategy 1, $J_1(,\delta,\tau,W' | S_1 = s_1, S_2 = s_2)$ is given by

$$J_1(\delta,\tau,W' | S_1 = s_1, S_2 = s_2) = \begin{cases} c_m\left[H(\tau)+H(W)-H(W')\right] & s_1, s_2 > W' \\ c_i(\delta)+c_m\left[H(\tau)+H_1(W')-H_1(s_1)\right] & W'-\tau < s_1 \le W', s_2 > W' \\ c_i(\delta)+c_m\left[H(\tau)+H_1(s_1+\tau)-H_1(s_1)+H_1(W)-H_1(W')\right] & \tau < s_1 \le W'-\tau, s_2 > W' \\ 2c_i(\delta)+c_m\left[H(\tau)+H_1(s_1+\tau)-H_1(s_1)+H_2(W)-H_2(s_2)\right] & \tau < s_1 \le W'-\tau, s_2 \le W' \end{cases} \tag{5}$$

where $H_2(t) = \int_0^t h_2(u)du$. In RHS of (5) , row 1, (rows 2 and 3), and row 4 coresponds to the expected warranty cost for 0, 1 and 2 imperfect repairs, respectively. Removing the conditioning in (5) yields

$$\begin{aligned} J_1(\delta,\tau,W') = {} & c_m\left[H(\tau)+H(W)-H(W')\right]\exp[H(\tau)-H(W')] + \\ & \int_{W'-\tau}^{W'} \left\{c_i(\delta)+c_m\left[H(\tau)+H_1(W)-H_1(s_1)\right]\right\} h(s_1)\exp[H(\tau)-H(s_1)]ds + \\ & \int_{\tau}^{W'-\tau} \left\{c_i(\delta)+c_m\left[H(\tau)+H_1(s_1+\tau)-H_1(s_1)+H_1(W)-H_1(W')\right]\right\} \\ & \exp[H_1(s_1+\tau)-H(W')]h(s_1)\exp[H(\tau)-H(s_1)]ds_1 + \\ & \int_{\tau}^{W'-\tau}\int_{s_1+\tau}^{W'} \left\{2c_i(\delta)+c_m\left[H(\tau)+H_1(s_1+\tau)-H_1(s_1)+H_2(W)-H_2(s_2)\right]\right\} \\ & h_1(s_2)\exp\left[H_1(\tau+s_1)-H_1(s_2)\right]h(s_1)\exp[H(\tau)-H(s_1)]ds_2ds_1 \end{aligned} \tag{6}$$

Using the similar approach as in Strategy 1 we have the expected warranty servicing cost for Strategy 2 $J_2(\delta,W_1,W_2,W_3)$ given by

$$\begin{aligned} J_2(\delta,W_1,W_2,W_3) = {} & c_m\left[H(W_1)+H(W)-H(W_3)\right]\exp[H(W_1)-H(W_3)] + \\ & \int_{W_2}^{W_3}\left\{c_i(\delta)+c_m\left[H(W_1)+H_1(W)-H_1(s_1)\right]\right\}h(s_1)\exp[H(W_1)-H(s_1)]ds_1 + \\ & \int_{W_1}^{W_2}\left\{c_i(\delta)+c_m\left[H(W_1)+H_1(W_2)-H_1(s_1)+H_1(W)-H_1(W_3)\right]\right\} \end{aligned}$$

$$\exp[H_1(W_2) - H_1(W_3)]h(s_1)\exp[H(W_1) - H(s_1)]ds_1 +$$
$$\int_{W_1}^{W_2}\int_{W_2}^{W_3}\left\{2c_i(\delta) + c_m\left[H(W_1) + H_1(W_2) - H_1(s_1) + H_2(W) - H_2(s_2)\right]\right\}$$
$$h_1(s_2)\exp[-H_1(s_2) + H_1(W_2)]h(s_1)\exp[H(W_1) - H(s_1)]ds_2 ds_1 . \tag{7}$$

Strategy 3

Let t_1 and t_2 denote the first and second imperfect maintenance, respectively. The first (second) imperfect maintenance may occur at either $t_1 = z_1$ if $Z_1 \le W_1$ or $t_1 = W_1$ if $Z_1 > W_1$ ($t_2 = z_2$ if $S_2 < Z_2 \le W_2$ or $t_2 = W_2$ if $Z_2 > W_2$). Then, there are four possible conditions of (t_1, t_2) given as follows.

$$t_1 = \begin{cases} z_1, t_2 = \begin{cases} z_2 & \text{if } S_1 < z_1 \le W_1 , S_2 < z_2 \le W_2 \\ W_2 & \text{if } S_1 < z_1 \le W_1 , z_2 > W_2 \end{cases} \\ W_1, t_2 = \begin{cases} z_2 & \text{if } z_1 > W_1 , S_2 < z_2 \le W_2 \\ W_2 & \text{if} z_1 > W_1 , z_2 > W_2 \end{cases} \end{cases} \tag{8}$$

Let $J_3(\delta, S_1, S_2, W_1, W_2 | Z_1 = z_1, Z_1 = z_2)$ be the expected warranty servicing cost conditional on Z_1 and Z_2. From (7) we have $J_3(\delta, S_1, S_2, W_1, W_2 | Z_1 = z_1, Z_1 = z_2)$ given by

$$J_3(\delta, S_1, S_2, W_1, W_2 | Z_1 = z_1, Z_1 = z_2) =$$

$$\begin{cases} 2c_{im}(\delta) + c_m(H(S_1) + H_1(S_2) - H_1(z_1) + H_2(W) - H_2(z_2)) & \text{if} \quad S_1 < z_1 \le W_1, S_2 < z_2 \le W_2 \\ c_{im}(\delta) + c_{ip}(\delta) + c_m(H(S_1) + H_1(S_2) - H_1(z_1) + H_2'(W) - H_2'(W_2)) & \text{if} \quad S_1 < z_1 \le W_1, z_2 > W_2 \\ c_{im}(\delta) + c_{ip}(\delta) + c_m(H(S_1) + H_1'(S_2) - H_1'(W_1) + H_2(W) - H_2(z_2)) & \text{if} \quad z_1 > W_1, S_2 < z_2 \le W_2 \\ 2c_{ip}(\delta) + c_m(H(S_1) + H_1'(S_2) - H_1'(W_1) + H_2'(W) - H_2'(W_2)) & \text{if} \quad z_1 > W_1, z_2 > W_2 \end{cases} \tag{9}$$

where $H_2(t) = \int_0^t h_2(u)du$ and $H_2'(t) = \int_0^t h_2'(u)du$. Removing the conditional form in (9) yields

$$J_3(\delta, S_1, S_2, W_1, W_2) = c_m H(S_1) + \int_{S_1}^{W_1}\int_{S_2}^{W_2}\left(2c_{im}(\delta) + c_m\left[H_1(S_2) - H_1(z_1) + H_2(W) - H_2(z_2)\right]\right)$$
$$h(z_1)\exp[H(S_1) - H(z_1)]h_1(z_2))\exp[H_1(S_2) - H_1(z_2)]dz_2 dz_1 +$$
$$\int_{S_1}^{W_1}\left(c_{im}(\delta) + c_{ip}(\delta) + c_m H_1(S_2) - H_1(z_1) + H_2'(W) - H_2'(W_2)\right)\exp[H_1(S_2) - H_1(W_2)]h(z_1)\exp[H(S_1) - H(z_1)]dz_1 +$$
$$\int_{S_2}^{W_2}\left(c_{im}(\delta) + c_{ip}(\delta) + c_m\left[H_1'(S_2) - H_1'(W_1) + H_2(W) - H_2(z_2)\right]\right)$$
$$\exp[H(S_1) - H(W_1)h_1'(z_2)\exp[H_1'(S_2) - H_1'(z_2)]dz_2 +$$
$$\left(2c_{ip}(\delta) + c_m(H_1'(S_2) - H_1'(W_1) + H_2'(W) - H_2'(W_2))\right)\exp[H(S_1) - H(W_1) \ \exp[H_1'(S_2) - H_1'(W_2). \tag{10}$$

3.2. Optimization

As it is very difficult to obtain the optimal solution analytically hence a numerical search will be used to find the optimal values τ^*, W'^* and δ^* for Strategy 1 and $\delta^*, S_1^*, S_2^*, W_1^*$ and W_2^* for Strategy 3 .

For Strategy 1, the optimization problem is $\min\limits_{\delta,\tau,W'} J_1(\delta,\tau,W')$ subject to the constraints $0 \le \tau \le W'/2$ and $0 \le W' \le W$, where $J_1(\delta,\tau,W')$ is given by the integral equation (6). Two stages are required to obtain the optimal values for τ^*, W'^* and δ^*.

Stage 1: For fixed τ and W_1, $\delta(\tau,W_1)$ is obtained by solving the following optimization problem:

$$MinJ_1(\delta|\tau,W_1) = \underset{\delta|\tau,W_1}{Min} \quad \Phi(\delta,\tau,W_1) = c_i(\delta)\Phi_1(\tau,W_1) + c_m\Phi_2(\delta,\tau,W_1) \tag{11}$$

where

$$\begin{aligned}\Phi_1(\tau,W_1) = &\int_{W_1-\tau}^{W_1} h(s_1)\exp[H(\tau)-H(s_1)]ds_1 + \int_{\tau}^{W_1-\tau} \exp[H_1(s_1+\tau)-H(W_1)]h(s_1)\exp[H(\tau)-H(s_1)]ds_1 \\ &+2\int_{\tau}^{W_1-\tau}\int_{s_1+\tau}^{W_1} h_1(s_2)\exp\left[H_1(\tau+s_1)-H_1(s_2)\right]h(s_1)\exp[H(\tau)-H(s_1)]ds_2ds_1\end{aligned} \tag{12}$$

$$\begin{aligned}\Phi_2(\delta,\tau,W_1) = &\int_{W_1-\tau}^{W_1}\begin{bmatrix}H(W-\delta s_1)\\ -H(s_1(1-\delta))\end{bmatrix}h(s_1)\exp[H(\tau)-H(s_1)]ds_1 \\ &+\int_{\tau}^{W_1-\tau}\begin{bmatrix}H(\tau)+H(s_1(1-\delta)+\tau)-H(s_1(1-\delta))\\ +H(W-\delta s_1)-H(W_1-\delta s_1)\end{bmatrix}\exp[H(s_1(1-\delta)+\tau-H(W_1)]h(s_1)\exp[H(\tau)-H(s_1)]ds_1 \\ &+\int_{\tau}^{W_1-\tau}\int_{s_1+\tau}^{W_1}\begin{bmatrix}H(\tau)+H(s_1(1-\delta)+\tau)-H(s_1(1-\delta))\\ +H(W-\delta s_1+\delta^2 s_1-\delta s_2)-H(s_2(1-\delta)-\delta s_1+\delta^2 s_1)\end{bmatrix} \\ &h(s_2-\delta s_1)\exp\left[H(\tau+s_1(1-\delta))-H(s_2-\delta s_1)\right]h(s_1)\exp\left[H(\tau)-H(s_1)\right]ds_2ds_1\end{aligned} \tag{13}$$

<u>Proposition</u>: If $\Phi_2(\delta,\tau,W_1)$ decreases in δ then there exists δ^* which minimizes the total cost of (11).

<u>Proof</u>:

Differentiating (11) partially with respect to δ and setting it to zero yields

$$\frac{\partial\Phi(\delta,\tau,W_1)}{\partial\delta} = p(c_p-c_m)\delta^{p-1}\Phi_1(\tau,W_1) + \frac{\partial}{\partial\delta}c_m\Phi_2(\delta,\tau,W_1) = 0$$

$$\frac{\partial}{\partial\delta}\Phi_2(\delta,\tau,W_1) = p(1-\frac{c_p}{c_m})\delta^{p-1}\Phi_1(\tau,W_1) \tag{14}$$

RHS of (14) is negative value as $(1-c_p/c_m)<0$, $(c_p/c_m)>1$, and all other components are positive values. This means that increasing in δ (improvement level) will decrease the sub total cost of minimal repair. With highier value of δ, the effect of imperfect repair gives more reduction in the age of the product, then this will lower the expected number of failures (and hence minimal repair) and the sub total cost of minimal repair. But on the other hand, as the first part of (11) $c_i(\delta)\Phi_1(\tau,W_1)$ is increasing function of δ then the sub total cost of imperfect repair increases with δ. As a result, there exists a value of δ that minimises the total cost of (11).

Stage 2: For a given δ^*, we obtain τ^* and W_1^* by solving the following optimization problem:

$$Min\ J_1(\tau,W_1|\delta) = \underset{\tau,W_1|\delta}{Min} \quad \Phi(\delta,\tau,W_1) = c_i(\delta)\Phi_a(\tau,W_1) + c_m\Phi_b(\delta,\tau,W_1)$$

subject to the constraint $0 \le \tau < W_1 \le W$. In this stage, a computational approach will be used to obtain the optimal parameter values.

For Strategies 2 and 3, the similar approach is used and also involves two stages as mentioned above. In the first stage, we seek δ^* for given $\{W_1, W_2, W_3\}$ or $[S_1, S_2, W_1, W_2]$ and then in the second stage we obtain $\{W_1^*, W_2^*, W_3^*\}$ or $[S_1^*, S_2^*, W_1^*, W_2^*]$ for given δ^*. Note that $\{...\}$ is a set of parameters for Strategy 2 and $[....]$ for Strategy 3.

4. NUMERICAL EXAMPLE

We consider $F(t)$ is given by Weibull distribution with two parameters - α and β being the scale and the shape parameters, respectively. The following nominal parameter values will be used: $\alpha = 3(year),\ \beta = 2,\ c_m = 1,$ and $W = 7$ years. The cost of imperfect repair is considered as a function of δ given by $c_{im}(\delta) = c_m + (c_p - c_m)\delta^4$ as in Yun et al. [4].

Results for Strategy 1:

Table 1 shows optimal solutions for Strategy 1 for a variety of c_p and a fixed value of δ.

Table 1. The optimal solutions for Strategy 1, c_p = 2, 4, ..., 10, and $\delta = 0.5$

c_p	$c_i(\delta)$	τ^*	W'^*	$J_1(\tau^*, W'^*)$
2	1.06	0.77	6.89	3.760
4	1.19	0.84	6.66	3.993
6	1.31	0.94	6.41	4.220
8	1.44	1.06	6.14	4.440
10	1.56	1.22	5.86	4.649

Remarks: τ^* increases and W'^* decreases as c_p increases. Meaning that increasing c_p (and hence $c_i(\delta)$) makes τ^* larger in order to gain a bigger benefits of imperfect repairs done.

Comparison of Strategies 1-2:

Now we compare the performances of Strategy 1 and Strategy 2 (developed by Varnosafaderani and Chukova [2]). Table 2 shows the effect of the scale parameter α (or the item reliability) and the length of the warranty period to the optimal servicing strategy (note that $c_m = 1$, $\beta = 2$, and $c_p = 6$). Strategy 2 is the best when the item reliability is low for each value of W. For the low reliability item, it is needed that the time between two imperfect repairs is less than τ and this in turn will give a better cost. But, if the reliability of the item is high representing by $\alpha > 2.0$, Strategy 1 is always the best strategy for $W, 3 \le W \le 7$, except that for W=7 and $\alpha = 2$ where Strategy 2 is the best.

Results for Strategy 3:

Table 3 shows optimal solutions for a variety of r ($=c_{ip}(\delta)/c_{im}(\delta)$) and $\alpha = 1(years),\ \beta = 2,\ c_m = 1,\ W = 7$ years. J_3^* decreases as r decreases. This shows that the cost of imperfect PM is getting smaller and S_1 (S_2) approaches W_1 (W_2) when r decreases. This is so as imperfect PM cost is cheaper than that of imperfect repair.

Table 2. The best strategy for α = 1.0,…,4.0 and W = 3,5,7

α	MTTF	W (year)		
		3	5	7
1.0	0.89	S-2	S-2	S-2
2.0	1.77	S-1	S-1	S-2
3.0	2.66	S-1	S-1	S-1
4.0	3.54	S-1	S-1	S-1

Table 3.The Optimal Solutions For Strategy 3 , $r = \{0.9, 0.7, 0.5\}$

r	$c_{ip}(\delta^*)$	$c_{im}(\delta^*)$	δ^*	S_1^*	W_1^*	S_2^*	W_2^*	J_3^*
0.9	3.18	3.53	0.84	2.27	3.48	4.35	5.54	25.111
0.7	3.36	4.80	0.93	2.39	2.39	4.61	4.61	24.530
0.5	3.00	6.00	1.00	2.33	2.33	4.67	4.67	22.333

Comparison of Strategies 1-3:

Table 4 shows the effect of α and W on the three strategies. For low reliability $(\alpha = 1, 2)$, Strategy 3 is the best for W=3 to 7, except for $\alpha = 2$ and W=3 Strategy 3 is the best. This indicates that for large W (W=5 and 7) PM is strongly requires to lower the expected warranty cost. For high reliability $(\alpha = 3, 4)$, Strategy 1 is the best for W=3 to 7. This means that the servicing strategy without PM is sufficiently enough to minimise the expected warranty cost. These sensitivity studies give a useful result for the manufacturer to select an effective servicing strategy when *W* and/or α change.

5. CONCLUSIONS

In this paper we have studied a servicing strategy for products sold with a long warranty period ($W \geq 3$ years). The servicing strategies studied involve imperfect repair and PM for a repairable product sold with a one-dimensional warranty. Imperfect repair done is dependent on the age of failure, which is different with that in Varnosafaderani and Chukova [1] based on sub-intervals formed.

Table 4. The best strategy for different α and W

α	MTTF	W (year)		
		3	5	7
1.0	0.89	S-3	S-3	S-3
2.0	1.77	S-1	S-3	S-3
3.0	2.66	S-1	S-1	S-1
4.0	3.54	S-1	S-1	S-1

Two strategies that have been developed (i.e. Strategies 1 and 3) give better results compared to Strategy 2 (proposed by [1]). Strategy 1 is always the best strategy for a high reliability whilst Strategy 3 for the low reliability. Moreover, Strategy 1 is easier to implement as it is only required to record the elapsed time since the last imperfect repair and τ (and hence a simple administrative work) in deciding an imperfect repair. Topics for future research are described as follows. The servicing strategy developed allows at most two imperfect repairs. The realistic servicing strategy is one which allows more than two imperfect repairs under warranty. The proposed strategy can be extended to two dimensional warranty case. Another topic that can be investigated is by introducing the servicing strategy to maintenance service contract model as proposed by [11] and [12]. These topics are currently under investigation.

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