

An Inspection Policy for a Repairable Item Sold With Warranty

¹Mustofa, A. Cakravastia, D. Irianto and BP. Iskandar

Graduate Program in Industrial Engineering and Management
Bandung Institute of Technology
Jl. Ganesa 10 Bandung, Indonesia 40132
¹Email: mustofa70@hotmail.com

Abstract- In general, a production system is unreliable in the sense that it will deteriorate due to the number of units produced and/or age. As a result, not all items produced are classified as conforming items-some items are nonconforming items. A nonconforming item is considered to have a shorter life time compared to that of the conforming item. If the item is sold with warranty of length W then the nonconforming item causes more failures under warranty. We assume that the item is in a defect state before it comes to a failed state. The state of an item (i.e. normal state, defect state, and failed state) can be detected by inspection and the item is repaired when it is in a defect state at inspection. Thus, failures due to the nonconforming item are minimized by carrying out inspection at time T ($< V$). In this paper we examine an inspection policy for an item sold with warranty. The policy is characterized by parameter T and we obtain T^* which minimizes the total cost (which is the sum of the preventive maintenance cost and the warranty cost). A numerical example is given to illustrate the optimal solution of the inspection policy.

Keywords: repairable, warranty product, inspection policy.

1. INTRODUCTION

In general, a production system is unreliable in the sense that it will deteriorate due to the number of units produced and/or age. As a result, not all items produced are classified as conforming item-some items are nonconforming items. A nonconforming item is considered to have a shorter life time compared to that of the conforming item. The nonconforming items often fail in early period of their life.

When the items are sold with warranty period W , the nonconforming items contribute more failures under warranty. As a consequence, the expected warranty servicing cost (warranty cost) associated with the nonconforming item is considerably higher than that of the conforming item. In a case, if the failure of an item results in a high cost consequence then the failure happened within warranty period causes a high warranty cost. The manufacturer has to reduce the number of the failures in order to decrease the warranty cost.

For an item where fails started with a defect state (potential failures) before it comes to a failed state, an inspection can be conducted to prevent the failures. The item detected in a defect state at inspection is preventively repaired for prolong its lifetime. On the other hand, each

inspection involves additional cost. The inspection conducted frequently will increase the total cost i.e. the sum of the warranty cost and the cost of the inspection (include the repair cost). The inspection is worthwhile for the manufacturer if the inspection cost needed is lower than the reducing of the warranty cost and in turn the inspection will give a cost saving. As a result, the manufacturer has to consider between the inspection cost and the reducing of the warranty cost resulted. A proper inspection procedure can minimize the total cost (maximize the cost saving). Therefore, it is important to find an optimal inspection procedure.

Optimal inspection policies developed to detect the defected state of an item have been studied by some researchers. (See Christer et al. (1984) and Wang and Christer (2003) in Zhao et al. (2009), Podofillini et al (2006) and Zhao et al (2007)). Scarf et al. (2009) have developed a hybrid of inspection and replacement policy. Zhao et al. (2009) develop an integrated methodology for optimizing inspection and maintenance of an item with considering delay repair. The study of the inspection for product sold with warranty can be found in Sandoh and Koide (2005).

Many researchers have discussed the preventive maintenance policy for products sold with warranty can be

found in Ritchken and Fuh (1986), Chun and Lee (1992), Chun (1992), Dagpunar and Jack (1994) and Yeh and Lo (2001). These papers deal with the periodic preventive maintenance within warranty period. In this paper, we propose an inspection policy that characterized by parameter T for a repairable item sold with warranty.

The outline of this paper is organized as follows. In section 2, we provide the notations to formulate the model. In section 3, we explain the details of the model formulation. In section 4, we analyze the model to find the optimal solution. In section 5, we give a numerical example to illustrate the performance of the model with inspection and compared to that of the model without inspection. Finally, Section 6 gives a brief conclusion and discussion for future work.

2. NOTATIONS

The following notations are used to formulate a mathematical model:

T	parameter of the policy inspection as a decision variable where $(0 < T < W)$
W	warranty period (day)
X	first time of the defect arrival of an item
Y	delay time
Z	first failure time of an item where $Z = X + Y$
p	proportion of the nonconforming item in a population where $0 < p < 1$
c_f	average cost of each failure repair
c_m	average cost of each minimal repair
c_s	average cost of each inspection
c_p	average cost of each preventive repair at inspection
$f(t), F(t), r(t)$	pdf, cdf and hazard rate for mixture
$f_n(t), F_n(t), r_n(t)$	pdf, cdf and hazard rate for the nonconforming items
$f_c(t), F_c(t), r_c(t)$	pdf, cdf and hazard rate for the conforming items
$f_X(t), F_X(t), r_X(t)$	pdf, cdf and hazard rate for the defect arrival of an item
$f_Y(t), F_Y(t), r_Y(t)$	pdf, cdf and hazard rate for the delay time
$J(T, W)$	expected total cost per unit sold as a function of T (\$/unit) with given W

3. MODEL FORMULATION

We consider repairable items sold with the warranty period W . Under this policy, the manufacturer has to rectify all claims at no cost to the buyer. We assume that all failures within the warranty period are valid claims and the time to inspect and repair is small and hence can be treated as being zero.

The item is one of three states i.e. normal state, defect

state and failed state. The state of the item can be detected perfectly by inspection. We assume that an item is in a defect state before it comes to a failed state. The item is inspected once time at time T during the warranty period.

3.1 Failure Modeling

We consider that the distribution of time to failure of the item follows a mixture distribution given by

$$F(t) = \gamma F_n(t) + (1 - \gamma) F_c(t) \quad (1)$$

The failure of the item is started by arrival of a defect before the item comes to failure denoted with X . The duration time from the arrival time of the defect to failure time is called delay time denoted Y (see Figure 1).

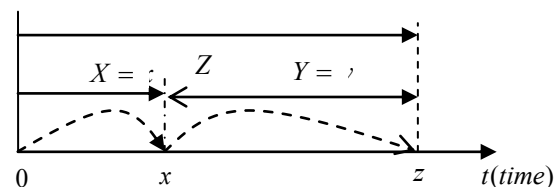


Figure 1. Failure process of the item

3.2 Inspection Policy

The inspection policy is as follows. It is assumed that a defective item is known only by inspection. The inspection is carried out one time at time T ($0 < T < W$) if only the item is never fail before. The inspection cost is denoted by c_s . There is no error in inspection and the inspection is non destructive. If the item is known in a defect state at inspection then the item is repaired preventively with repair cost c_p ($> c_s$). After that, the item would become a conforming item.

3.3 Servicing Strategy

We consider the different servicing of the nonconforming and conforming items. The failure of the nonconforming items are rectified by general repair whereas after the general repair, it will restore the nonconforming item into a conforming item. The failure of the conforming items are always rectified by minimal repair. We assume that the nonconforming items are showed by a first failure incurred within the warranty period. All the first failure of the items are generally repaired with repair cost c_f ($> c_p$) under no preventive repair done at inspection. Furthermore, the subsequent failures are minimally repaired with repair cost c_m .

Suppose that $Z_1 = \tau < W$ is the first failure time then the process of subsequent failures in $(\tau, W]$ following the non-homogeneous Poisson process. The expected number of the failures within warranty period is based on Z_1 . Since $Z_1 = X_1 + Y_1$ then Z_1 is based on X_1 and Y_1 . Suppose that $X_1 = x$ and $Y_1 = y$ then the expected number of the failures in $(x+y, W]$, $R_c(W - \tau - y)$, is given by

$$R_c(W - x - y) = \int_{x+y}^W \lambda_c(t - x - y) dt \quad (2)$$

From Equation (2), the expected number of the failures within the warranty period is $1 + \lambda_c(W - \tau - y)$ (see Fig.2).

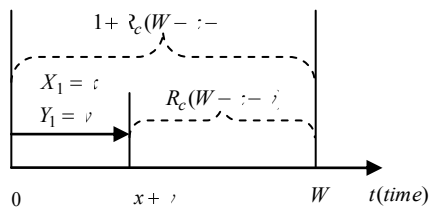


Figure 2. The expected number of the failures with conditional on $X_1 = x$ and $Y_1 = y$

4. MODEL ANALYSIS

4.1 Warranty Cost

We obtain the expected warranty cost by using a conditional approach. Let $J(T | X_1 = x, Y_1 = y)$ denotes the expected warranty cost based on the servicing strategy conditional on $X_1 = x$ and $Y_1 = y$. $J(T | X_1 = x, Y_1 = y)$ is given by

$$J(T; W | X_1 = x, Y_1 = y) = \begin{cases} c_f + c_m R_c(W - x - y) & \text{if } x < T, y < T - x \\ c_s + c_p + c_m R_c(W - T) & \text{if } x < T, y > T - x \\ c_s + c_f + c_m R_c(W - x - y) & \text{if } T < x < W, y < W - x \\ c_s & \text{if } T < x < W, y > W - x \\ c_s & \text{if } x > W \end{cases} \quad (3)$$

By removing the conditioning, we have

$$J(T; W) = \int_0^T \int_0^{T-x} (c_f + c_m R_c(W - x - y)) f_Y(y) f_X(x) dy dx + \int_0^T (c_s + c_p + c_m R_c(W - T)) (1 - F_Y(T - x)) f_X(x) dx + \int_T^{W-x} \int_0^{W-x} (c_s + c_f + c_m R_c(W - x - y)) f_Y(y) f_X(x) dy dx + \int_T^W c_s (1 - F_Y(W - x)) f_X(x) dx + c_s (1 - F_X(W)) \quad (4)$$

4.2 Optimization

We seek the optimal parameter value T that minimizes $J(T; W)$ given by (4). As $J(T; W)$ involves a complex integral equation then we use a computational approach for obtaining the optimal value of T .

5. NUMERICAL EXAMPLE

We consider that distribution function of failure time follows Weibull distribution given by

$$F(t) = p \left[1 - \exp \left[- \left(\frac{t}{\alpha} \right)^\beta \right] \right] + (1 - p) \left[1 - \exp \left[- \left(\frac{t}{\alpha} \right)^\beta \right] \right] \quad (5)$$

where α is the scale parameter and β is the shape parameter. We assume that the parameter values are $\alpha = 100, \beta = 2; \alpha = 300, \beta = 5; W = 600$ days, $c_f = 0.5, c_p = 1, c_s = 2, c_m = 2$. Furthermore, those values are used as the base parameter values in the numerical examples.

In this section, we first show the optimal T for various values of α, β (see Table 1). Secondly, we investigate the effect of the values of p to the model.

Table 1. The optimal T for various values of

α, β at $p = 0.05$				T (day)	$J(T; W)$ (\$/unit)
Nonconforming		Conforming			
α	β	α	β		
100	2	200	5	200	8.797
100	2	300	5	301	6.183
200	2	300	5	301	6.097
200	2	400	5	308	3.047
250	2	400	5	308	2.985
250	2	500	5	309	1.925

In Table 1, we find that the values of α, β influence the optimal T and the expected total cost. The increasing of α, β will decrease the expected total cost more significant compared to that of the increasing of α, β .

Figure 3 shows the effect of the values of p to the model performance for $p = \{0.01, 0.02, 0.03, 0.04, 0.05\}$. In Figure 3, we found that for $0.01 < p < 0.05$, the optimal T is not affected. The variability of p affects to the expected total cost. The increasing of p would increase the expected total cost.

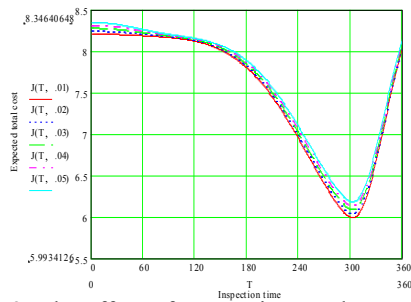


Figure 3. The effect of proportion on the expected total cost

For fixed c_s, c_p and c_m , we will show the effect of the increasing of c_f (see Figure 4). The result shows that the optimal T is unaffected with the increasing of c_f . Furthermore, we obtain that for increasing failure cost the effect of the inspection policy in reducing the warranty cost is more significant.

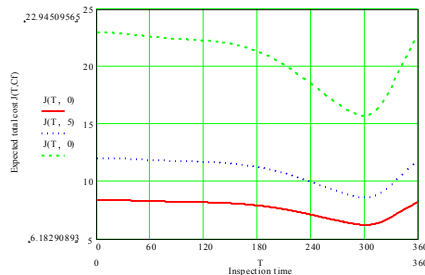


Figure 4. The effect of failure cost on the expected cost for $p = .05$

The nonconforming item has been considered to have a shorter life than the item conforming item. In Figure 3 and 4, the mean time to failure (MTTF) of the nonconforming item is around 89 days and the conforming item is around 276 days. Furthermore, we increase the MTTF of the nonconforming item by shifting the value of α to 150,200,250 and 300. The effects of the variability of α are shown in Figure 5. We obtain that for increasing α the expected total will decrease.

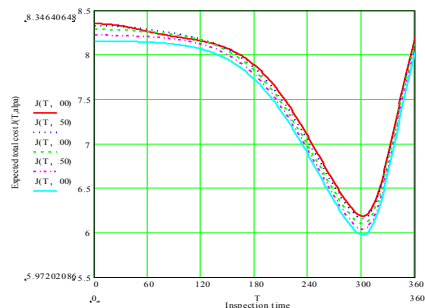


Figure 5. The effect of scale parameter α on the expected total cost for $p = .05, c_f = 0$

6. CONCLUSION

In this paper we have studied an inspection policy to reduce the warranty cost for populations which have a nonconforming item. The inspection is carried out to prevent the failure of the nonconforming item. The policy reduce the warranty cost significantly, especially when the failure cost is costly and/or dangerous. This inspection policy is developed for repairable items. The policy can be developed for non-repairable items in the next research.

REFERENCE

Chun Y.H Optimal number of periodic preventive maintenance operations under warranty, *Reliability Engineering and System Safety*. 37. pp. 223-225

Dagpunar J.S and Jack N (1994) Preventive maintenance strategy for equipment under warranty. *Microelectronic Reliability*, Vol. 34, No. 6 pp 1089-1093

Podofilini, L, Zio, E. and Vatn, J. Risk-informed optimization of railway tracks inspection and maintenance procedures. *Reliability Engineering and System Safety*, 2006, No. 91, pp.20-35

Scarf P.A., Cavalcante C.A., Dwight R.A and Gordon P. (2009) An age-based inspection and replacement policy for heterogeneous components, *IEEE Transaction on Reliability*, Vol. 58 No. 4

Sandoh, H. and Koide, K. (2005) Comparison between two scale-inspection and warranty policies for weight-quality. *International Journal of Quality & Reliability Management*, Vol.22 No.7, pp.731-748

Zhao, J., Miao, S., and Zhang, S. (2009) An Integrated Model for Optimizing Maintenance and Inspection of Potential Failures, *Reliability, Maintainability and Safety*, pp. 626-629

Zhao, J., Chan, A.H. and Roberts, C. (2007) Reliability Evaluation and Optimization for Non-Homogeneous Systems under imperfect Inspection. *Reliability Engineering and System Safety*, Vol. 92, No.1, pp. 65-73.

Yeh, R.H. and Lo H-C (2001) Optimal preventive-maintenance warranty policy for repairable products, *European Journal of Operational Research* 134 pp. 59-69

AUTHOR BIOGRAPHIES

Mustofa is a Ph.D student in Faculty of Industrial Technology, ITB Bandung. His dissertation is reliability and warranty area.

Andi Cakravastia is an Assistant Professor in Department of Industrial Engineering, Faculty of Industrial Technology,

Bandung Institute of Technology, Indonesia. His teaching and research interests include supply chain management and applied operations research.

Dradjat Irianto is an Associate Professor in Department of Industrial Engineering, Bandung Institute of Technology, Indonesia. His teaching and research interests include quality engineering, process quality control, and the implementation of quality management.

B.P. Iskandar is a Professor at Faculty of Industrial Technology, ITB his research interests include Product warranties, Warranty claim Data, and Reliability and Maintenance models.