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Servicing Strategy and Preventive Maintenance for Products Sold with One-Dimensional Warranties

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Abstract- **A manufacturer sells its product with a longer warranty period in order to increase the product competitiveness. Offering a product with a longer warranty period increases the warranty cost to the manufacturer. For a repairable product, an appropriate servicing strategy can reduce the warranty cost significantly. Many servicing strategies involving replacement or imperfect repair have been studied in the literature. In this paper, we study a servicing strategy which considers preventive maintenance and imperfect repair to reduce the warranty cost.**

Keywords- **servicing strategy, preventive maintenance, one-dimensional warranty, imperfect repair**

I. INTRODUCTION

Nowadays, a lot of manufacturers tend to offer longer warranty periods in order to increase their product competitiveness. For example products of electronics and automotives are sold with warranty period ranging from 3 to 7 years. Offering a product with a longer warranty period increases the warranty cost to the manufacturer and this becomes a major interest to reduce it. For a repairable product, an appropriate servicing strategy can significantly reduce the warranty cost.

An optimal servicing strategy (repair-replace strategy) for products sold with a one-dimensional warranty has been proposed by [1] but this strategy is difficult to implement. Reference [2] proposed an alternative servicing strategy which is easy to implement in practice. In this strategy, the warranty period is divided into three intervals called interval I, II and III. The first failure in interval II is replaced with a new one and all other failures are minimally repaired.

When the cost of the replacement is very high compared to the cost of repair then the servicing strategy involving replacement in [2] is not economical. The servicing strategy similar to [2] but using imperfect repair instead of replacement has been developed for one-dimensional warranties by [3]. Reference [4] developed model as in [3] for two-dimensional warranties.

The servicing strategies in [3] and [4] only allow one imperfect repair over the warranty period. For a longer warranty period, more imperfect repairs would be needed in order to reduce the number of failures over the warranty period. Reference [5] studied a servicing strategy allowing more than one replacement for one-dimensional warranties. Further, reference [6] developed a servicing strategy involving more than one imperfect repair for products sold with two-dimensional warranties. An alternative servicing strategy for one dimensional warranty, which allows more than one imperfect repair over the warranty period is proposed by [7]. In this strategy, imperfect repairs are carried out based on the age at failure.

Reference [8] proposed a preventive maintenance (PM) where imperfect PM is carried out at scheduled times and all failures occurring before the scheduled PM are fixed by minimal repair. In this paper, we extend the strategy in [7] to incorporate PM as in [8]. In other words, the proposed strategy integrates servicing strategy and PM to reduce the warranty cost. This strategy can be applied for warranted products where PM is offered one package with the warranty.

The outline of the paper is as follows. Section 2 describes of the model formulation. Section 3 deals with the model analysis. In section 4, we give a numerical example to illustrate the optimal solution and compare the results of the proposed strategy with those reported in the literature. Finally, we conclude with a brief discussion for further research.

II. MODEL FORMULATION

A. Notation

The following notation will be used in model formulation.

W	: warranty period (in year)
δ	: improvement level ($0<\delta<1$)
W'	: parameter of Strategy 1 ($0<W'\le W$)
τ	: threshold time of Strategy 1 ($0<\tau\le W'/2$)
S_1, S_2, W_1, W_2	: parameters of Strategy 2 ($0\le S_1\le W_1\le S_2\le W_2\le W$)
$F(t)$, $f(t)$, $h(t)$	: failure distribution, density and hazard rate functions
$A_i(t)$	: virtual age after i^{th} imperfect ($i=1,2$)
Z_1	: random variable of first failure after τ (S_1) for Strategy 1 (2) with distribution function $F_1(z_1)$
Z_2	: random variable first failure after $Z_1+\tau$ (S_2) for Strategy 1 (2) with distribution function $F_2(z_2)$

c_m	: cost of minimal repair
c_p	: cost of perfect repair
$c_{im}(\delta)$	: cost of imperfect repair as a function of δ
$c_{ip}(\delta)$	: cost of imperfect PM as a function of δ
$J_1(\delta,\tau,W')$	: expected warranty servicing cost of Strategy 1
$J_2(\delta,S_1,S_2,W_1,W_2)$	: expected warranty servicing cost of Strategy 2

B. Integrating Servicing Strategy and Preventive Maintenance

We consider a repairable product sold with a one-dimensional free repair warranty with the warranty period W. Two servicing strategies-namely Strategy 1 and Strategy 2 will be studied. Strategy 1 has been studied in [7] and Strategy 2 is the proposed strategy.

Strategy 1:

Imperfect repair is done at failure (at time $t, 0 < t \le W' \le W$) if the elapsed time since the last imperfect repair (or the beginning of the operation, $t = 0$) is greater than τ (a threshold value). All other failures are fixed by minimal repair. As a result, this servicing strategy allows more than one imperfect repair.

Strategy 2:

The warranty period is divided by five intervals i.e. $(0,S_1]$, $(S_1,W_1]$, $(W_1,S_2]$, $(S_2,W_2]$ and $(W_2,W]$. Each first failure in $(S_1,W_1]$ or $(S_2,W_2]$ is imperfectly repair and all other failures are minimally repaired. When there is no failure in interval $(S_1,W_1]$ ($(S_2,W_2]$) then PM is done at W_1 (W_2). (Note: all imperfect repair in Strategy 1 and 2 are done at improvement level δ). Strategy 2 can be viewed as the extention of Strategy 1, which incorporates PM into the strategy or a more general servicing strategy.

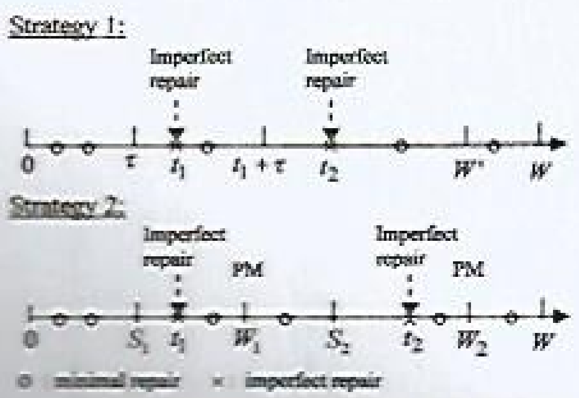

Fig. 1. Strategies 1 and 2 with two imperfect repairs

C. Effect of Imperfect Maintenances

It is assumed that imperfect PM and imperfect repair improve the reliability of the product in the sense that the age of the item after repair is smaller than that before failure. The imperfect PM (imperfect repair) will reduce the age of the product with improvement level δ_1 (δ_2). Without losing the generality of improvement levels, it is considered that $\delta_1 = \delta_2 = \delta$.

The effect of imperfect maintenance can be modelled through either hazard rate or age reduction models [9]. In this paper the effect of the imperfect maintenances is modelled by reducing the virtual age of the product. Let $A(t)$ be the virtual age of the product at time t. The hazard rate of the product is as function of $A(t)$, denoted by $h(A(t))$. If the imperfect maintenance is done at age t_1 with improvement level δ then the virtual age and the hazard rate after repair are given by $A_1(t) = t - \delta t_1$ and $h_1(t) = h(t - \delta t_1)$ for $t > t_1$. For the second imperfect maintenance occurring at age t_2 the virtual age and hazard rate are given by $A_2(t) = t - \delta t_1 + \delta^2 t_1 - \delta t_2$ and $h_2(t) = h(t - \delta t_1 + \delta^2 t_1 - \delta t_2)$ for $t > t_2$, respectively. For the case where all failures are rectified by minimal repair then $A(t)$ is equal to t that is called actual age.

We assume that the cost of imperfect PM is smaller than that of imperfect repair under the same improvement level. The cost of imperfect PM is given by

$$c_{ip}(\delta) = r c_{im}(\delta) \qquad 0 < r < 1 \qquad (1)$$

where r is the cost ratio between the imperfect PM cost and the imperfect repair cost.

D. Modeling Failures for Imperfect Maintenance

We consider the case where the number of imperfect maintenances over the warranty period is at most two times. Let Z_1 and Z_2 denote the first failure after S_1 and S_2, respectively. The distribution functions for Z_1 and Z_2 [$F_1(z_1)$ and $F_2(z_2)$] are given as follows. As failures occurring in $(0,S_1]$ and $(t_1,S_2]$ are fixed by minimal repair, then

$$F_1(z_1) = 1 - \exp[H(S_1) - H(z_1)] \qquad (2)$$

where $H(t) = \int_0^t h(u)du$.

Differentiating (2) with respect to z_1 yields

$$f_1(z_1) = h(z_1)\exp[H(S_1) - H(z_1)]. \qquad (3)$$

Conditioning on $Z_1 = z_1$ and then unconditioning it, we have $F_2(z_2)$ given by

$$F_2(z_2) = \int_{S_1}^{W_1} \{1 - \exp[H_1(S_2) - H_1(z_2)]\} h(z_1) \exp[H(S_1) - H(z_1)] dz_1 + \{1 - \exp[H_1'(S_2) - H_1'(z_2)]\} \exp[H(S_1) - H(W_1)] \quad (4)$$

where $H_1(t) = \int_0^t h_1(u)du$ and $H_1'(t) = \int_0^t h_1'(u)du$ and its density function is given by

$$f_2(z_2) = \int_{S_1}^{W_1} \{h_1(z_2) \exp[H_1(S_2) - H_1(z_2)]\} h(z_1) \exp[H(S_1) - H(z_1)] dz_1 dz_1 + h_1'(z_2) \exp[H_1'(S_2) - H_1'(z_2)] \exp[H(S_1) - H(W_1)]. \quad (5)$$

Distribution functions of Z_1 and Z_2 for Strategy 1 can be found in [7].

III. MODEL ANALYSIS

In the strategy developed, we consider the case where the number of imperfect maintenances during the warranty period is at most N times, where $N = 2$.

A. Expected Warranty Servicing Cost

We obtain the expected warranty cost for Strategies 1 and 2 by using the conditional approach.

Strategy 1

Using conditional approach similar in [5] we obtain the expression for the expected warranty servicing cost for Strategy 1 given by

$$\begin{aligned} J_1(\delta, \tau, W^*) = {} & c_m[H(\tau) + H(W) - H(W^*)] \exp[H(\tau) - H(W^*)] + \\ & \int_{W^*-\tau}^{W^*} \{c_{im}(\delta) + c_m[H(\tau) + H_1(W) - H_1(z_1)]\} h(z_1) \exp[H(\tau) - H(z_1)] dz_1 \\ & \int_{\tau}^{W^*} \{c_{im}(\delta) + c_m[H(\tau) + H_1(z_1 + \tau) - H_1(z_1) + H_1(W) - H_1(W^*)]\} \\ & \exp[H_1(z_1 + \tau) - H(W^*)] h(z_1) \exp[H(\tau) - H(z_1)] dz_1 + \\ & \int_{\tau}^{W^*-\tau} \int_{z_1+\tau}^{W^*} [2c_{im}(\delta) + c_m[H(\tau) + H_1(z_1 + \tau) - H_1(z_1) + \\ & H_2(W) - H_2(z_2)]] h(z_2) \exp[H_1(\tau + z_1) - H_1(z_2)] \\ & h(z_1) \exp[H(\tau) - H(z_1)] dz_2 dz_1 \end{aligned} \quad (6)$$

Strategy 2

Let t_1 and t_2 are first and second imperfect maintenance, respectively. The first (second) imperfect maintenance may occur at either $t_1 = z_1$ if $Z_1 \le W_1$ or $t_1 = W_1$ if $Z_1 > W_1$ ($t_2 = z_2$ if $S_2 < Z_2 \le W_2$ or $t_2 = W_2$ if $Z_2 > W_2$). Then, there are four possible conditions of (t_1, t_2) given as follows.

$$t_1 = \begin{cases} z_1, t_2 = \begin{cases} z_2 & \text{if } S_1 < z_1 \le W_1, S_2 < z_2 \le W_2 \\ W_2 & \text{if } S_1 < z_1 \le W_1, z_2 > W_2 \end{cases} \\ W_1, t_2 = \begin{cases} z_2 & \text{if } z_1 > W_1, S_2 < z_2 \le W_2 \\ W_2 & \text{if } z_1 > W_1, z_2 > W_2 \end{cases} \end{cases} \quad (7)$$

Let $J_2(\delta, S_1, S_2, W_1, W_2 | Z_1 = z_1, Z_1 = z_2)$ be the expected warranty servicing cost conditional on Z_1 and Z_2. From (7) we have $J_2(\delta, S_1, S_2, W_1, W_2 | Z_1 = z_1, Z_1 = z_2)$ given by

$$J_2(\delta, S_1, S_2, W_1, W_2 | Z_1 = z_1, Z_1 = z_2) = \begin{cases} 2c_{im}(\delta) + c_m(H(S_1) + H_1(S_2) - H_1(z_1) + H_2(W) - H_2(z_2)) & \text{if } S_1 < z_1 \le W_1, S_2 < z_2 \le W_2 \\ c_{im}(\delta) + c_{ip}(\delta) + c_m(H(S_1) + H_1(S_2) - H_1(z_1) + H_2'(W) - H_2'(W_2)) & \text{if } S_1 < z_1 \le W_1, z_2 > W_2 \\ c_{im}(\delta) + c_{ip}(\delta) + c_m(H(S_1) + H_1'(S_2) - H_1'(W_1) + H_2(W) - H_2(z_2)) & \text{if } z_1 > W_1, S_2 < z_2 \le W_2 \\ 2c_{ip}(\delta) + c_m(H(S_1) + H_1'(S_2) - H_1'(W_1) + H_2'(W) - H_2'(W_2)) & \text{if } z_1 > W_1, z_2 > W_2 \end{cases} \quad (8)$$

where $H_2(t) = \int_0^t h_2(u)du$ and $H_2'(t) = \int_0^t h_2'(u)du$.

Removing the conditional form in (7) yields

$$\begin{aligned} J_2(\delta, S_1, S_2, W_1, W_2) = {} & c_m H(S_1) + \int_{S_1}^{W_1} \int_{S_2}^{W_2} (2c_{im}(\delta) + c_m[H_1(S_2) - H_1(z_1) + H_2(W) - H_2(z_2)]) \\ & h(z_1) \exp[H(S_1) - H(z_1)] h_1(z_2)) \exp[H_1(S_2) - H_1(z_2)] dz_2 dz_1 + \\ & \int_{S_1}^{W_1} (c_{im}(\delta) + c_{ip}(\delta) + c_m H_1(S_2) - H_1(z_1) + H_2'(W) - H_2'(W_2)) \\ & \exp[H_1(S_2) - H_1(W_2)] h(z_1) \exp[H(S_1) - H(z_1)] dz_1 + \\ & \int_{S_2}^{W_2} (c_{im}(\delta) + c_{ip}(\delta) + c_m[H_1'(S_2) - H_1'(W_1) + H_2(W) - H_2(z_2)]) \\ & \exp[H(S_1) - H(W_1) h_1'(z_2) \exp[H_1'(S_2) - H_1'(z_2)] dz_2 + \\ & (2c_{ip}(\delta) + c_m(H_1'(S_2) - H_1'(W_1) + H_2'(W) - H_2'(W_2))) \exp[H(S_1) - H(W_1)) \\ & \exp[H_1'(S_2) - H_1'(W_2). \end{aligned} \quad (9)$$

B. Optimization

As it is very difficult to obtain the optimal solution analytically hence a numerical search will be used to obtain the optimal values τ^*, W^{r*} and δ^* for Strategy 1 and $\delta^*, S_1^*, S_2^*, W_1^*$ and W_2^* for Strategy 2 .

IV. NUMERICAL EXAMPLE

We consider that $F(t)$ is given by Weibull distribution with two parameters - α and β (representing scale and the shape parameters, respectively). The following nominal parameter values will be used: $\alpha = 1(years)$, $\beta = 2$, $c_m = 1$, and $W = 7$ years. The cost of imperfect repair is considered as a function of δ given by $c_{im}(\delta) = c_m + (c_p - c_m)\delta^4$ as in [3]. Table 1 shows optimal solutions for Strategy 1 for a variety of r ($= c_{ip}(\delta)/c_{im}(\delta)$).

TABLE 1
THE OPTIMAL SOLUTIONS FOR STRATEGY 2 , $r = (0.9, 0.7, 0.5)$

r	$c_{ip}(\delta^*)$	$c_{im}(\delta^*)$	δ^*	S_1^*	W_1^*	S_2^*	W_2^*	J_2^*
0.9	3.18	3.53	0.84	2.27	3.48	4.35	5.54	25.111
0.7	3.36	4.80	0.95	2.39	2.39	4.61	4.61	24.530
0.5	3.00	6.00	1.00	2.33	2.33	4.67	4.67	22.333

$J_2^* = J_2(\delta^*, S_1^*, S_2^*, W_1^*, W_2^*)$

Remarks: J_2^* decreases as r decreases. Decreasing r shows the cost of imperfect PM is getting smaller. S_1 (S_2) approaches W_1 (W_2) when r decreases. This is so as imperfect PM cost is cheaper than that of imperfect repair.

Now we compare the performances of Strategy 1 and 2 for a various values of α with the strategy developed by [6], which shows the effect of α or reliability of the product to the performances of those strategies. In this comparison, the strategy developed as in [6] is called Strategy 3. The expected warranty servicing cost of Strategy 3 is given by $J_3(\delta, W_1, W_2, W_3)$. Table 2 shows the results for a variety of α at $r = 0.5$ (Note: MTTF is mean time to first failure).

TABLE 2
RESULTS OF STRATEGY 1, 2 AND 3

α	MTTF	J_1^*	J_2^*	J_3^*
1.0	0.89	25.196	23.333	25.111
2.0	1.77	8.201	8.055	8.147
3.0	2.66	4.175	4.189	4.205
4.0	3.54	2.999	2.752	2.596

[illegible]

$\alpha = 1$ and $\alpha = 4$ represents the lowest and highest product reliability, respectively. Table 2 shows that Strategy 2 (proposed strategy) is the best strategy for the product with low reliability ($\alpha = 1$ and $\alpha = 2$). But Strategy 1 is the best for the high reliability. As a result, the proposed strategy (Strategy. 2) becomes the best strategy if the cost of PM is relatively small and the reliability of the product is low. When PM has not been considered, Strategy 3 is the best strategy for products with low reliability.

V. CONCLUSION

In this paper we have studied a servicing strategy which incorporates PM for products sold with a long warranty period ($W \geq 3$ years).

The strategy can be extend to two dimensional warranty case and this topic is currently under investigation.

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